University of Basel

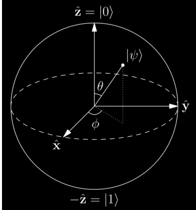
THE UNIVERSITY OF CHICAGO

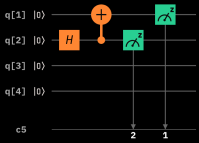
Pritzker School of Molecular Engineering

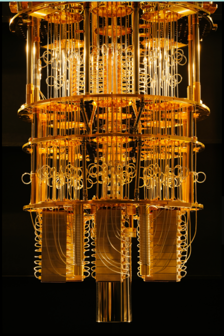
Physical Implementation of Quantum Computing

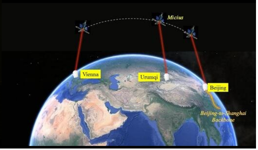
Martin Koppenhöfer

https://www.quantumtheory-bruder.physik.unibas.ch/









General outline

previous session

- introduction into quantum computing

this session

- which quantum-computing platforms exist?
- what are their benefits / drawbacks?
- how does one operate a quantum computer?
- **web-based** access on IBM's quantum computers

tomorrow's session

- programming IBM's quantum computers with **python**

Online resources

`https://www.quantumtheory-bruder.physik.unibas.ch/people/martin-koppenhoefer/quantum-computing-and-robotic-science-workshop.html`

- installation guide for tomorrow's session
- material for tomorrow's session (will be uploaded later today)
- slides

Outline of this session

1. Recap
2. What are properties of a good qubit?
3. Overview of quantum-computing platforms
4. Nuclear magnetic resonance (NMR)
5. Ions in electromagnetic traps
6. Electron spins in semiconductor quantum dots
7. Superconducting electrical circuits
8. Comparison of quantum-computing platforms
9. Operating a quantum processor
10. Cloud-based access

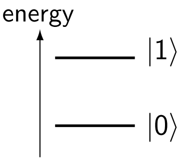
Recap

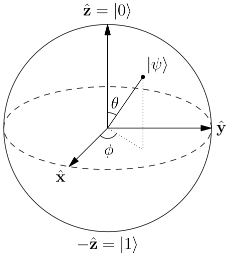
What are quantum bits (qubits)?

- a classical computer manipulates bits:
  possible states 0 or 1 are *discrete*
- a quantum computer manipulates qubits
  $\equiv$ *quantum two-level systems*:

  possible states $(\alpha|0\rangle + \beta|1\rangle)$ are *continuous*

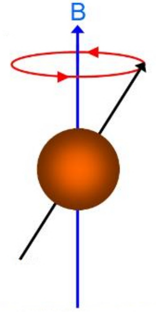
  α, β are complex numbers, $|\alpha|^2 + |\beta|^2 = 1$.
- measuring a qubit yields a classical bit (probabilistically)
- state of a qubit can be represented as a vector, e.g., in the z basis
  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



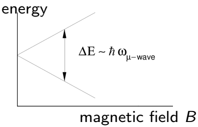


Recap

Example: electron in a magnetic field

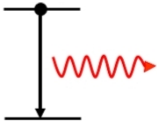


- spin: quantum-mechanical property
- two states: $|\uparrow\rangle$ and $|\downarrow\rangle$
- states $|\uparrow\rangle$ and $|\downarrow\rangle$ have different energy in a magnetic field B
  - spin-up state $|\uparrow\rangle \rightarrow$ logical state $|0\rangle$
  - spin-down state $|\downarrow\rangle \rightarrow$ logical state $|1\rangle$
- applying magnetic fields at the level-splitting energy ΔE generates transitions $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$



Recap

Decoherence



Transition of a quantum state to a classical state

$$|\psi\rangle = |\alpha|\,|0\rangle + e^{i\phi}\sqrt{1-|\alpha|^2}\,|1\rangle$$

- Relaxation: Qubit makes the irreversible transition

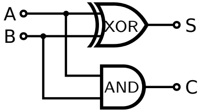
  $$|1\rangle \to |0\rangle$$

  on average: exponential decay with half-life period T_1
- Dephasing: Qubit loses the superposition
  on average: half-life period T_2

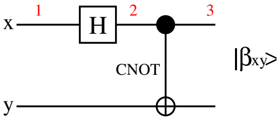
Recap

What is a quantum algorithm?

- a classical computer manipulates a N-bit state by logical gates
- algorithms are built from a universal set of gates (e.g., the NAND gate)



- a quantum computer manipulates a N-qubit state by quantum gates
- algorithms are built from a universal set of quantum gates (e.g. H, T, and CNOT)

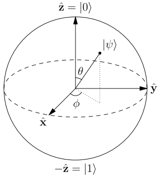


Recap

Single-qubit gates

- NOT gate: $\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\hat{X}|0\rangle = |1\rangle$$
$$\hat{X}|1\rangle = |0\rangle$$



- Hadamard gate: $\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Recap

Controlled NOT gate (CNOT)

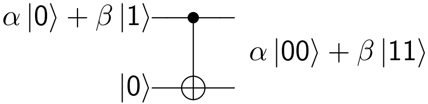
- 2-qubit gate
- flip second (target) qubit if first (control) qubit is $|1\rangle$:

$$\begin{aligned} |00\rangle &\to |00\rangle \\ |01\rangle &\to |01\rangle \\ |10\rangle &\to |11\rangle \\ |11\rangle &\to |10\rangle \end{aligned} \qquad \mathrm{CNOT} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

- circuit diagram:

control qubit:

target qubit:



How to build a quantum computer

- identify a two-level system as a part of a larger physical system
- find out how to initialize the qubit
- find out how to do gates and measurements

What are properties of a good qubit?

Di-Vincenzo criteria

Properties of a good qubit:

1. scalability: build a large (e.g., 10^9) number of qubits
2. initialization: prepare a well-defined initial quantum state
3. long coherence time: in comparison to the gate time
4. universal set of quantum gates: to construct all possible quantum gates
5. measurement procedure: to get the result of a calculation

Overview of quantum-computing platforms

- spins in large molecules + NMR
- ions in electromagnetic traps
- neutral atoms in optical lattices
- optical quantum computing
- ^{31}P donor atoms in silicon
- electron spins in semiconductor quantum dots
- superconducting electrical circuits
  - flux qubit
  - charge qubit
  - phase qubit
  - transmon qubit
- topological qubits

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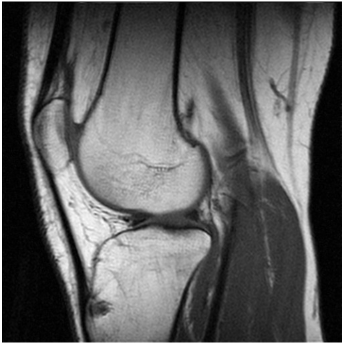
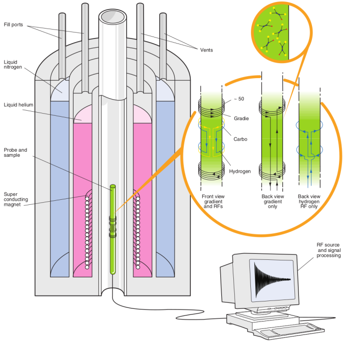
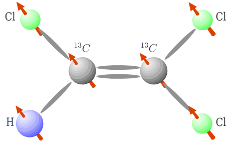
Nuclear magnetic resonance (NMR)



Nuclear magnetic resonance (NMR)



Trichloroethylene: 3 qubits



- use nuclei with spin $\frac{1}{2}$
- apply static magnetic field B to split the states $|\uparrow\rangle$ and $|\downarrow\rangle$
- single-qubit gates: radio-frequency magnetic fields
- multi-qubit gates: use interaction of spins within one molecule
- readout: precessing spins induce voltage in readout coils
- Shor's prime-factoring algorithm demonstrated on 7 qubits

[Laflamme *et al.*, arXiv:quant-ph.0207172 (2002)]

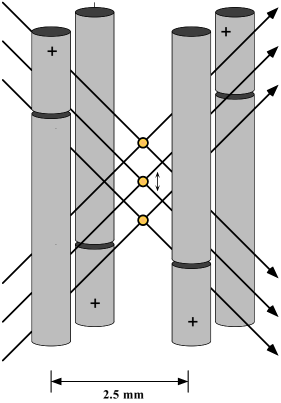
Nuclear magnetic resonance (NMR)

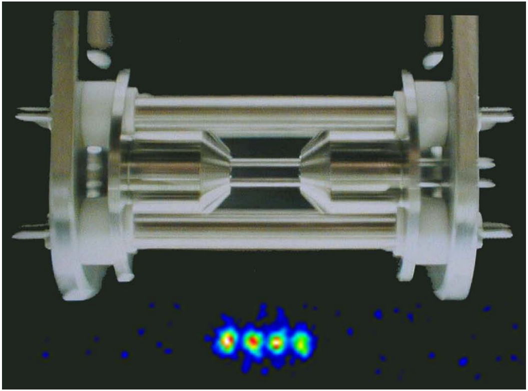
Di-Vincenzo benchmark

- scalability: ✗
- initialization: ✓
- long coherence time: ✓
- universal set of quantum gates: ✓
- measurement procedure: ✓

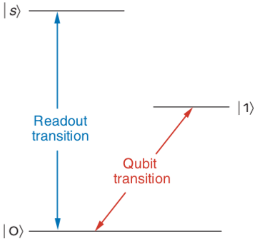
Ions in electromagnetic traps

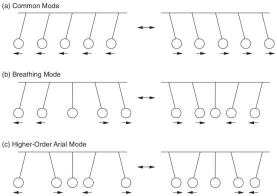
- $\lesssim 50$ ions (e.g., ^{9}Be, ^{40}Ca) in harmonic electromagnetic trap





Ions in electromagnetic traps





- qubit encoded in two long-lived internal states $|0\rangle$, $|1\rangle$ of an ion
- single-qubit gates: laser beams induce transitions $|0\rangle \leftrightarrow |1\rangle$
- readout: drive transition from $|0\rangle$ to short-lived state $|s\rangle$, detect photon emitted during relaxation $|s\rangle \rightarrow |0\rangle$.
- multi-qubit gates: ions repel each other

  $\Rightarrow$ oscillation modes along the chain

  $\Rightarrow$ useful for qubit-qubit interaction

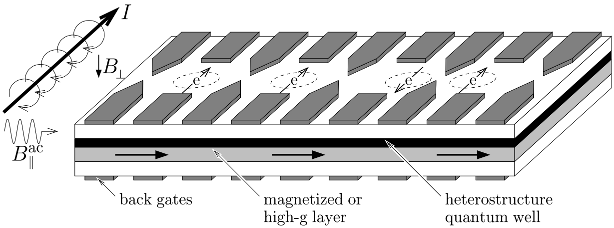
[M. Holzscheiter, Los Alamos Science **27**, 264 (2002)]

Ions in electromagnetic traps

Di-Vincenzo benchmark

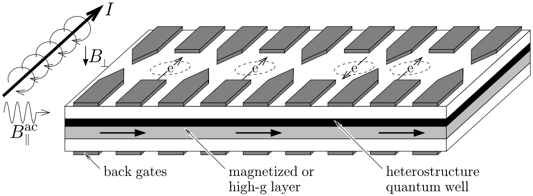
- scalability: ✗
- initialization: ✓
- long coherence time: ✓
- universal set of quantum gates: ✓
- measurement procedure: ✓

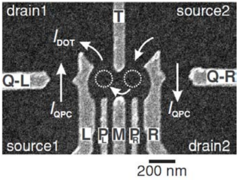
Electron spins in semiconductor quantum dots



- A two-dimensional electron gas (2DEG) can be realized in semiconductor heterostructures
- 2DEG can be structured by gate electrodes (negative potential repels electron gas under the electrode)
- quantum dots may be formed which contain a small number or only a single electron

Electron spins in semiconductor quantum dots





- $B_\perp$ splits the states $|\uparrow\rangle$, $|\downarrow\rangle$
- single-qubit gates: apply time-dependent magnetic field $B_\parallel(t)$
- two-qubit gates using exchange interaction between spins of neighboring dots $\hat{H}_{\rm ex} = \sum_{\langle i,j\rangle} J_{ij}\hat{S}_i \cdot \hat{S}_j$
  coupling strength J_{ij} depends on gate voltages
- readout: single-electron transistor or quantum point contact

Electron spins in semiconductor quantum dots

Di-Vincenzo benchmark

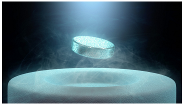
- scalability: ✓ (semiconductor technology!)
- initialization: ✓
- long coherence time: ✓
- universal set of quantum gates: ✓
- measurement procedure: ✓

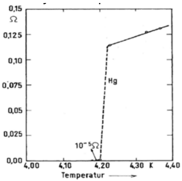
Superconducting electrical circuits

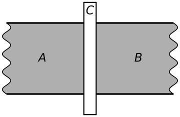
What is a superconductor?

- macroscopic quantum systems with zero resistance below a critical temperature T_c
- electrons form Cooper pairs characterized by a macroscopic wavefunction $\Psi = \sqrt{n_s} e^{i\varphi}$
- two superconductors (A, B) separated by an insulating oxide barrier (C) form a Josephson junction
- Josephson effect: even in the absence of a voltage across the Josephson junction, a supercurrent I can flow:

$$I = I_\mathrm{c} \sin(\varphi_\mathrm{left} - \varphi_\mathrm{right})$$
$$\hat{H} = -E_J \cos(\varphi_\mathrm{left} - \varphi_\mathrm{right})$$

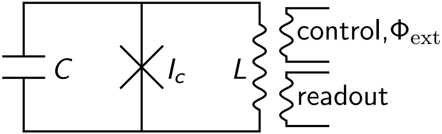


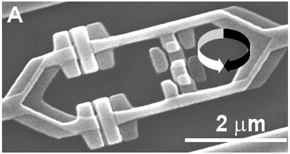




Superconducting electrical circuits

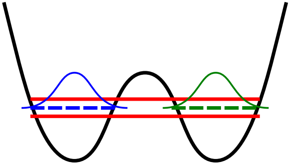
Flux and phase qubit



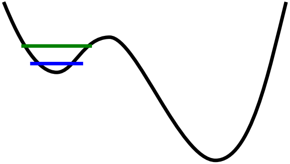


$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos(\hat{\varphi}) - \frac{(\hat{\varphi} - \Phi_{\text{ext}})^2}{2L}$$

Flux qubit: $\Phi_{\text{ext}} = \frac{\Phi_0}{2}$
superpositions of $|\circlearrowleft\rangle$ and $|\circlearrowright\rangle$

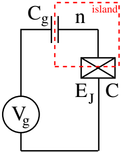


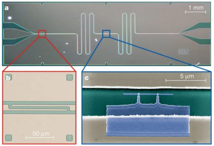
Phase qubit: $\Phi_{\text{ext}} \approx \Phi_0 = \frac{h}{2e}$
states $|0\rangle$ and $|1\rangle$ in same well



Superconducting electrical circuits

Charge and transmon qubit

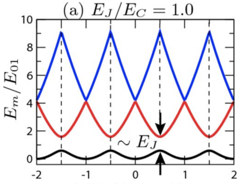




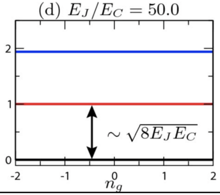
control & readout

$$\hat{H} = E_C(\hat{n} - n_g)^2 - E_J \cos(\hat{\varphi})$$

Charge qubit: $E_C \gg E_J$
superpositions of 0 or 1 Cooper pairs on the island

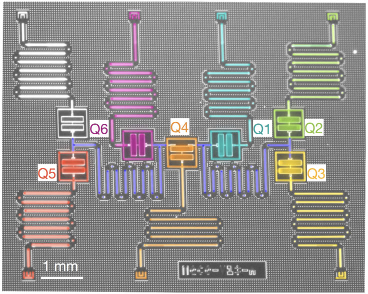


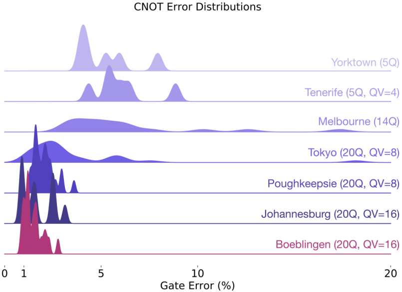
Transmon: $E_C \ll E_J$
Lowest eigenstates in an anharmonic potential



Superconducting electrical circuits

Charge and transmon qubit





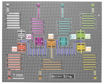
- seven fixed-frequency transmon qubits
- each qubit is coupled to a microwave resonator to do single-qubit gates and readout
- several qubits are coupled by microwave resonators to implement multi-qubit gates

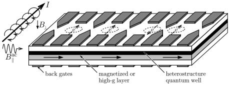
Superconducting electrical circuits

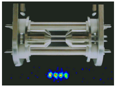
Di-Vincenzo benchmark

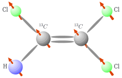
- scalability: ✓
- initialization: ✓
- long coherence time: ✓
- universal set of quantum gates: ✓
- measurement procedure: ✓

Comparison of quantum-computing platforms









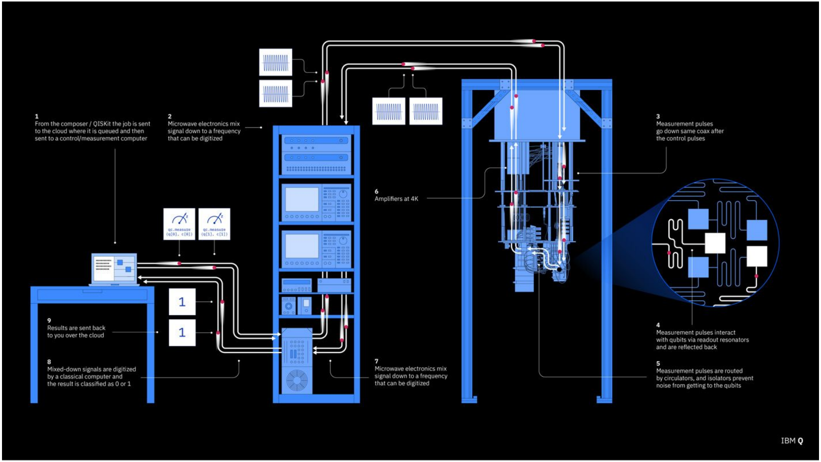
	superconducting qubit	electron spin qubit	trapped ions	NMR
footprint	$\approx$ µm	0.1 µm	spacing 10 µm	mm
scalability	yes	yes	complicated	no
energy gap	$1 - 20$ GHz	$1 - 10$ GHz	$10^5 - 10^6$ GHz	MHz
temperature	10 mK	100 mK	µK	300 K
single-qubit gate time τ_1	$\approx$ ns	10 ns	µs	ms
two-qubit gate time τ_2	$10 - 50$ ns	0.2 µs	100 µs	10 ms
coherence time T_2	$10 - 100$ µs	ms $-$ s	0.1 s	10 s
1-qubit gate fidelity (%)	98 — 99.9	98 — 99.9	99.1 — 99.9999	98 — 99
2-qubit gate fidelity (%)	96 — 99.4	89 — 96	97 — 99.9	98
initialization	yes	yes	yes	ensemble
readout fidelity (%)	99	97	99.99	ensemble

[Xiang et al., Rev. Mod. Phys. **85**, 623 (2013)]
[Resch et al., arXiv:1905.07240 (2019)]
[Keith et al., Phys. Rev. X **9**, 041003 (2019)]

$$5\,\mathrm{GHz} \approx 250\,\mathrm{mK}$$

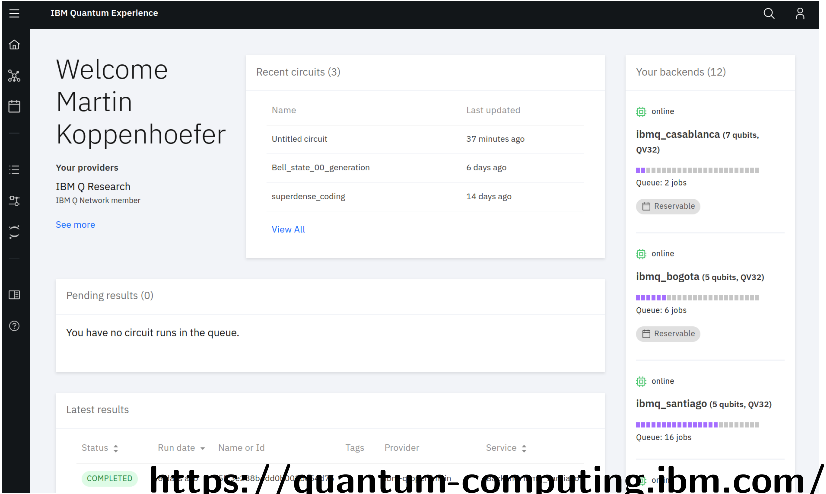
Operating a quantum processor

The lab around a quantum processor



Cloud-based access

The dashboard



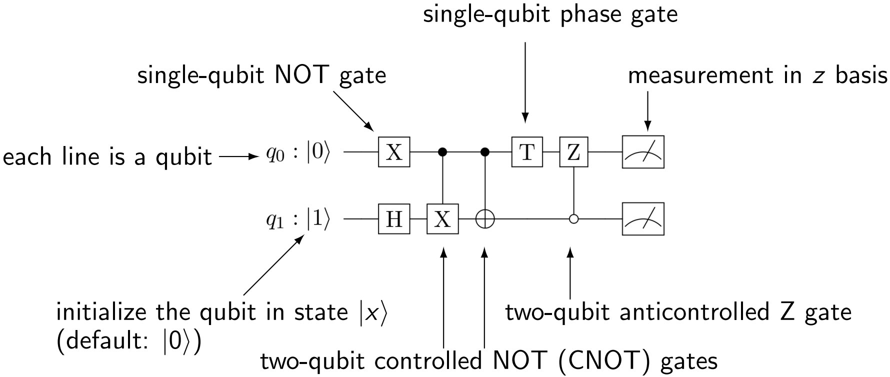
https://quantum-computing.ibm.com/

Cloud-based access

A quantum processor

Cloud-based access

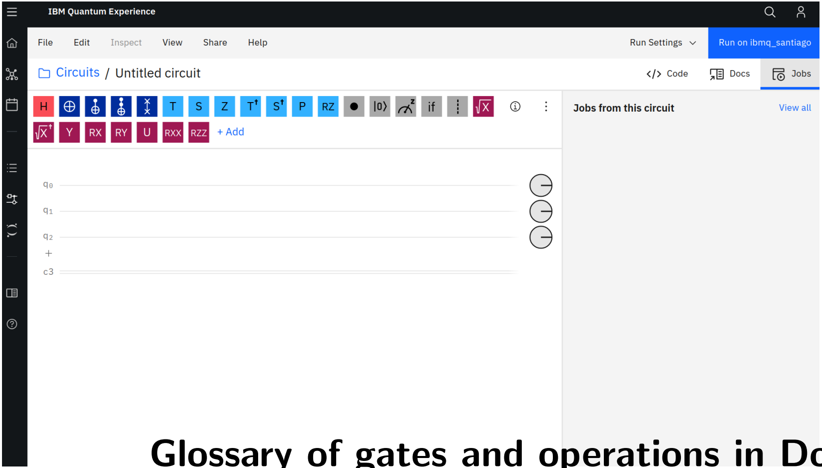
Quantum circuits



$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Cloud-based access

The circuit editor



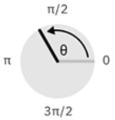
Glossary of gates and operations in Docs

Cloud-based access

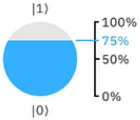
Visualizing the state of a single qubit

$$\sqrt{1 - |\alpha|}\,|0\rangle + e^{i\theta}\,|\alpha|\,|1\rangle$$

- relative phase θ of a superposition



- probability $|\alpha|^2$ to measure the $|1\rangle$ state

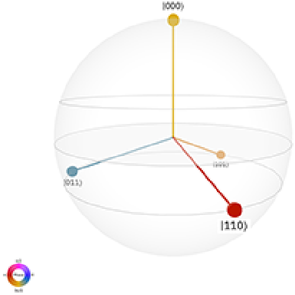


- entanglement

Cloud-based access

Visualizing the state of a quantum register

$$\begin{aligned}
&\alpha_{000}\,|000\rangle\\
&+\alpha_{100}\,|100\rangle+\alpha_{010}\,|010\rangle+\alpha_{001}\,|001\rangle\\
&+\alpha_{110}\,|110\rangle+\alpha_{101}\,|101\rangle+\alpha_{011}\,|011\rangle\\
&+\alpha_{111}\,|111\rangle
\end{aligned}$$



- layer n contains all states with n qubits in state $|1\rangle$
- thickness of node of state $|x\rangle$: $|\alpha_x|$
- color of node of state $|x\rangle$: $\arg(\alpha_x)$

Generating Bell states

Quantum circuit

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

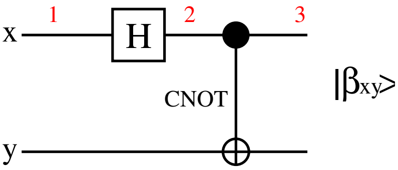
$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

General expression:

$$\boxed{|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}(|0y\rangle + (-1)^x |1\bar{y}\rangle)}$$



1. input state: $|xy\rangle = |00\rangle$
2. apply Hadamard gate $\hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$: $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$
3. apply CNOT gate: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\beta_{00}\rangle$

Generating Bell states

Reminder: Entanglement

- Bell states are a crucial resource for quantum algorithms because they are entangled
- classical N-bit states can be "factorized"

Example: classical state (11)

- bit 1 is in state "1", bit 2 is in state "1"
- equivalent quantum state: $|11\rangle = |1\rangle \otimes |1\rangle$

- But there are quantum states that cannot be factorized

Example: state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq |\psi_1\rangle \otimes |\psi\rangle_2$

- system can only be described as a whole

Generating Bell states

Reminder: Entanglement

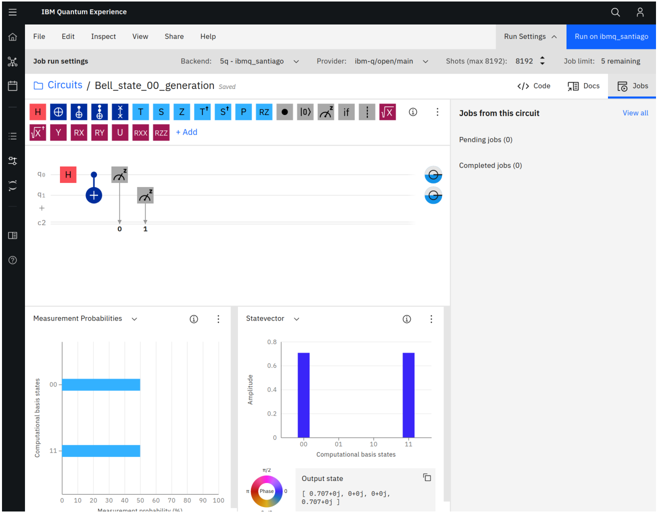
- entanglement $\Rightarrow$ correlations

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

- What happens if we measure qubit 1 and 2 in the z basis?
- either we get 0 for qubit 1 and 0 for qubit 2 (probability $\frac{1}{2}$)
- or we get 1 for qubit 1 and 1 for qubit 2 (probability $\frac{1}{2}$)
- but never any "mixed" result

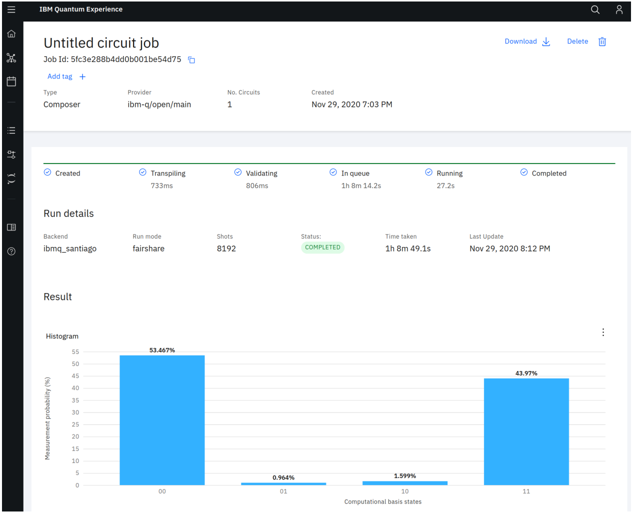
Generating Bell states

Online demonstration



Generating Bell states

Online demonstration



Educational material

Start your path towards learning *Quantum Algorithms*

https://qiskit.org/learn

Qiskit textbook

Youtube series *Coding with Qiskit*

Online course *Introduction to QC*

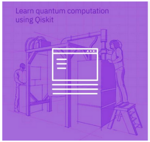
Learning resources

The below are designed and created by the Qiskit team. However, we recommend a familiarity with linear algebra and Python from these trusted resources.

All resources | Beginner | Advanced

Time to spend learning

- (•) any
- ( ) 1 minute
- ( ) 1 day
- ( ) 1 week
- ( ) 1 month
- ( ) 1 year

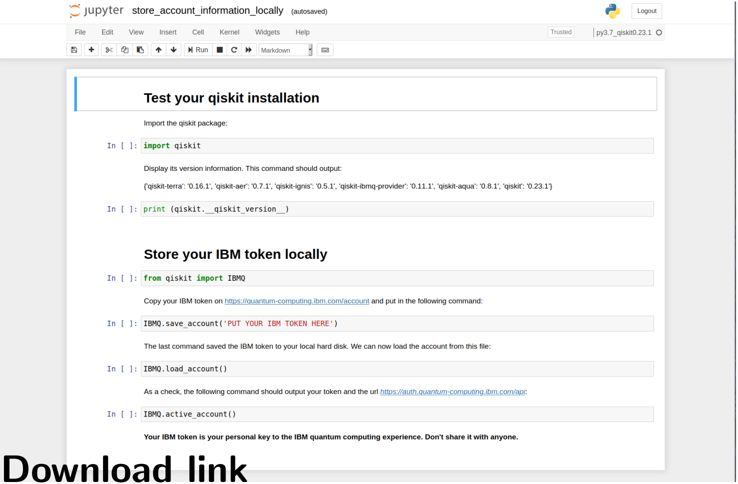


Qiskit Textbook

The Qiskit Textbook is a free digital open source textbook that will teach you the concepts of quantum computing while you learn to use Qiskit.

Read the textbook

Material for next session



Download link

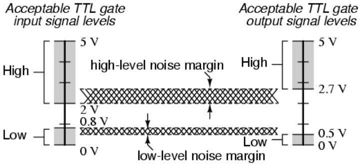
University of Basel

Thank you

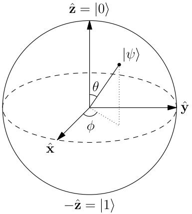
for your attention.

Backup slides

Discrete vs. continuous states



[https://www.allaboutcircuits.com/]



consider $\hat{H}_\varepsilon = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 + i\varepsilon \\ 1 & -1 - i\varepsilon \end{pmatrix}$:

$$\hat{H}\hat{H}\,|0\rangle = |0\rangle$$

$$\hat{H}_\varepsilon\hat{H}_\varepsilon\,|0\rangle = \left(1 + i\frac{\varepsilon}{2}\right)|0\rangle - i\frac{\varepsilon}{2}\,|1\rangle$$